ABSTRACT

of dissertation for the Philosophy Doctor (PhD) degree in specialty "6D060400 – Physics" by **Sagidullayeva Zhanna** on the topic
"**LOCAL AND NON-LOCAL INTEGRABLE SPIN SYSTEMS WITH SELF-CONSISTENT POTENTIALS**"

The thesis is devoted to the study of some integrable local and nonlocal generalized spin models within the framework of soliton theory.

Relevance of the dissertation theme

Currently, the theory of solitons is rapidly developing. A soliton is a structurally stable solitary wave propagating in a nonlinear medium. Due to their special properties, solitons behave like particles (particle-like wave): when interacting with each other or with some other disturbances, they are not destroyed, but continue to move, keeping their structure unchanged. This property can be used to transmit data over long distances without interference. This opens up enormous possibilities for the use of solitons. Solitons and soliton-like waves can be observed in various fields of physics. The mathematical description of solitons has also been developed. Over the past decades, the theory of nonlinear evolutionary equations has advanced so much that nonlocal models have emerged: spatial, temporal, and spatiotemporal.

The study of spin systems is one of the fundamental problems of modern physics. However, questions related to the description of nonlocal integrable spin systems remain open, and this direction has become one of the active areas of research in the field of physics and mathematics in recent years. Within the framework of soliton theory, the definition of interactions of spins and spin chains quite succinctly includes the theory of integrable systems. An important subclass of nonlinear evolution equations - integrable spin systems that describe the dynamics of propagation of wave packets in magnets - includes the generalized Landau-Lifshitz equation. This study examines generalized Landau-Lifshitz equation with self-consistent potentials and its equivalents. A fundamental understanding of spin systems, as well as systems with self-consistent potentials, and their study will help expand our understanding of magnetic materials and spin dynamics. This will make it possible to study various aspects such as the structure of spin waves, magnetic domains and interactions between spins, which is important for the development of new materials and technologies based on magnetism.

Integrable spin systems with self-consistent potentials are of great interest from the point of view of theoretical physics. They make it possible to obtain accurate analytical solutions, as well as to study the properties of solitons - nonlinear wave structures that can be stable and move without distortion. The study of solitons in spin systems is of significant importance for the development of new methods for transmitting and processing information. Research in this area contributes to the development of spintronics and magnetoelectronics. Spintronics is a multidisciplinary field of research and involves scientists and researchers from various fields such as physics, materials science, electronics and technology. These areas of technology are

based on the use of properties and control of the spin momentum of electrons. Understanding and controlling spin dynamics in various systems opens up new opportunities for creating more efficient and powerful devices, such as magnetic memory devices, spintronic transistors and logic gates, magnetic sensors, etc.

The results of research in the field of spin systems with self-consistent potentials can have direct application in various technological fields. For example, the development of new materials and structures based on these systems can lead to the creation of more efficient and powerful devices in the fields of spintronics, magnetoelectronics and information technology. In addition, it contributes to the development of fundamental aspects of mathematical physics and has potential for application in other fields of science and engineering.

The purpose of the research

Theoretical study of local and nonlocal integrable spin systems with potential, search for soliton solutions.

To achieve the goal, it was necessary to solve **the following tasks**:

1. Describe the appearance of solitons in ferromagnetic materials using the example of one-dimensional magnets.

2. Find soliton solutions of the generalized Heisenberg equation.

3. Obtain a new spin system, gauge equivalent to the equation of the family of nonlinear Schrödinger equations, its Lax representation for local and nonlocal cases.

Object of research

Spin waves and spin dynamics in ferromagnetic materials.

The subject of research

Soliton and soliton-like solutions that correctly describe the dynamics of spin interaction in ferromagnetic materials, including the condition of the presence of potential.

Research methods

The dissertation work used methods of soliton theory, such as the direct Hirota method, the Darboux transform method and extended $\left(\frac{G'}{G^2}\right)$- expansion method for constructing solutions of equation with both constant and variable coefficients. The above methods are extremely optimal for finding exact and analytical solutions of integrable spin systems. They also make it possible to develop a systematic construction of different types of solutions: soliton, solutions such as rogue waves, breathers, domain walls, etc.

Scientific novelty of the work

The scientific novelty of the dissertation work lies in the fact that a new spin model has been obtained that describes the movement of the magnetization vector in ferromagnets. Studying the behavior of the magnetization vector and spin waves allows us to formulate a hypothesis about the possibility of the appearance of solitons and soliton-like formations in a given medium. In particular, the following results were obtained:

• A new spin system that is gauge equivalent to a family of nonlinear Schrödinger-type equations.

• Lax representation, which proves the integrability of the resulting spin model

within the framework of soliton theory.

• Exact soliton and soliton-like types of solutions of the nonlocal generalized spin model and model with potential.

• Generalization of methods for finding soliton and soliton-like solutions of nonlocal spin models. Non-local versions of the considered models given in the work can provide further extensions with complex components. The paper shows how nonlocality can be naturally included in a pair of auxiliary equations that arise in this situation. How nonlocality is implemented in standard types of procedures for solving nonlinear equations.

These results contribute to the development of methods in the theory of solitons, studies of integrable spin systems, as well as the expansion of approaches related to the presence of *P*- and *PT*-symmetries of the models under consideration.

Provisions for the defense

1. The coefficient a in the generalized Landau-Lifshitz equation with a self-consistent potential affects the behavior of the components of the spin vector S_3 and potential W_3; with an increase in this coefficient, an increase in the amplitude of the potential is observed.

2. The resulting new spin system, gauge-equivalent to a generalized equation of the type of nonlinear Schrödinger equations, describes the movement of the magnetization vector in ferromagnets; in the nonlocal case the spin matrix S is not Hermitian and has $PT-$symmetry of the form $S(t,x)=\sigma_3 S^{\dagger}(t,-x)\sigma_3$, and in the local case it S is Hermitian.

3. The extended generalized $\left(\frac{G'}{G^2}\right)$-expansion method allows to find solutions to nonlinear evolution equations with both constant and variable coefficients, while the basic $\left(\frac{G'}{G^2}\right)$-expansion method can only be used with constant coefficients.

The theoretical and practical significance of the research outcomes

This dissertation relates to theoretical research. The results obtained make it possible to characterize the processes describing spin waves in one-dimensional ferromagnets and can lead to important applications in spintronics. The results obtained can be used in studying the dynamics of spins in ferromagnetic materials, as well as in teaching specialized disciplines in higher educational institutions.

The personal contribution of the author

The main results of the dissertation research were obtained personally by the author. Numerical calculations and graphs of the solutions found were carried out by the author independently. Publications on the topic of the dissertation were prepared by the author personally. Problem setting, selection of research methods and discussion of results were carried out jointly with scientific consultants.

Approbation of the dissertation

The main research results obtained in the dissertation were presented at the following conferences:

1. The International Conference in Advancement in Mathematical Sciences (AMS-2015) Antalya, Turkey 2015.

2. The International Scientific Conference “Modern problems of applied mathematics and information technology – Al-Khorezmiy 2016” Bukhara, Uzbekistan 2016.

3. The first Annual Meeting of Kazakh Physical Society, Nazarbayev University, Astana 2018.

4. Seminars of the scientific interdepartmental research group “Functional analysis and mathematical physics” (Department of Physics, California State University, Fresno).

5. International Conference New Trends in the Applications of Differential Equations in Sciences, St. Constantine and Helena, Bulgaria 2019.

6. The XXVIth International Conference on Integrated Systems and Quantum symmetries (ISQS-26) Prague, Czech Republic 2019.

7. “Ufa Autumn Mathematical School - 2021”, Bashkir State University, Ufa, Bashkortostan, Russia.

8. The XII. International Symposium on Quantum Theory and Symmetries (QTS12) Prague, Czech Republic.

In addition, the results obtained were discussed at the Department of General and Theoretical Physics of the L.N. Gumilyov Eurasian National University and at seminars of the Eurasian International Centre for Theoretical Physics.

Publications

Based on the results of the dissertation work, 19 articles were published, including: 2 articles in journals with a non-zero impact factor indexed in the international information resources Web of Science and Scopus; 3 articles in collections of foreign international conferences, included in the Web of Science and Scopus databases, 3 articles in periodicals recommended by Committee for Quality Assurance in Science and Higher Education of the Republic of Kazakhstan; 11 articles and theses in materials of international conferences of the Republic of Kazakhstan. Main publications:

1. Sagidullayeva Zh., Nugmanova G., Myrzakulov R., Serikbayev N., Integrated Kuralay Equations: Geometry, Solutions and Generalizations // SYMMETRY-BASEL. – 2022. – Vol. 14, No. 7. – P. 1374. Impact Factor 2022 – 2.7. Q2, Percentile 78.

2. Mohanty S. K., Pradhan B., Sagidullayeva Zh., Myrzakulov R., Dev A. N., Exact solutions for the Bogoyavlensky-Konopelchenko equation with variable coefficients with an efficient technique // Alexandria Engineering Journal. –2023.– Vol. 72. – P. 287-293. Impact Factor 2022 – 6.8. Q1, Percentile 93.

3. Nugmanova G., Myrzakulov R., Sagidullayeva Zh. Hirota's method for a spin model with self-consistent potential // Journal of Physics: Conference Series, 804, 2017. – 012035, Percentile 22.

4. Sagidullayeva, Z., Nugmanova, G., Myrzakulov, R. Integrated surfaces induced by generalized Landau-Lifshitz equation with self-consistent potential // Journal of Physics: Conference Series, 2019, Percentile 22.

5. Sagidullayeva, Z., Nugmanova, G., Myrzakulov, R. Dispersionless limit of the Heisenberg ferromagnet equation with self-consistent potential // AIP Conference Proceedings, 2019, Percentile 15.

6. Nugmanova G.N., Sagidullayeva Zh.M. Generalized spin model with vector potential and its solution // Bulletin of Karaganda University, Series Mathematics. - Karaganda, 2017.-№2(86).-P. 91-96.

7. Sagidullayeva Zh.M. On the gauge equivalence of the two-layer M-XCIX equation and the two-component Schrödinger-Maxwell-Bloch equation // Bulletin of the L.N. Gumilyov Eurasian National University. – Series physics, astronomy. – Astana, 2018 ¬№3(124).¬ P.41-46.

8. Albatyrova M.B., Sagidullayeva Zh.M. Rogue wave and soliton solution for the nonlinear integrable spin model // Reports of the National Academy of Sciences of the Republic of Kazakhstan. – 2023. – Vol. 2., No. 346. – P. 19-26.

Relation of the dissertation topic with the plans of scientific works

The work was carried out in accordance with research plans within the framework of grant funding projects for the following projects:

1. 0893/GF4 “Investigation of the generalized Landau-Lifshitz equation with self-consistent sources and its integrable reductions” 2015-2017 (supervisor G.N. Nugmanova).

2. AP08857372 “Investigation of the connection between the geometry of surfaces/manifolds and integrable nonlinear evolution equations” 2020-2022. (supervisor Nugmanova G.N.).

3. AR14972423 “Investigation of some integrable local and non-local generalized spin systems” (grant funding for research of young scientists under the project “Zhas Galym” for 2022-2024) supervisor Sagidullayeva Zh.M.

Scope and structure of the dissertation

The dissertation consists of an introduction, four chapters, a conclusion and a list of sources used. The work is presented on 90 pages of typewritten text, contains 22 drawings and graphs, 279 formulas are given, the list of sources used contains 104 titles.